Process Sequences

Recurrent Neural Networks

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A lot of the ideas in this lecture come from Andrej Karpathy's blog post on the Unreasonable Effectiveness of RNNs (`http://karpathy.github.io/2015/05/21/rnn-effectiveness/`). Many of the images and animations were made by Adam Prügel-Bennett.

Recurrent Neural Networks - Motivation

x:	Jon	and	Antonia	gave	deep	learning	lectures
y:	1	0	1	0	0	0	0

Recurrent Neural Networks - Motivation

x:	$x^{(1)}$	...	$x^{(t)}$	...	$x^{(T_x)}$
x:	Jon	...	Antonia	...	lectures
y:	$y^{(1)}$	...	$y^{(t)}$	...	$y^{(T_y)}$
y:	1	...	1	...	0

In this example, $T_x = T_y = 7$ but T_x and T_y can be different.

Recurrent Neural Networks

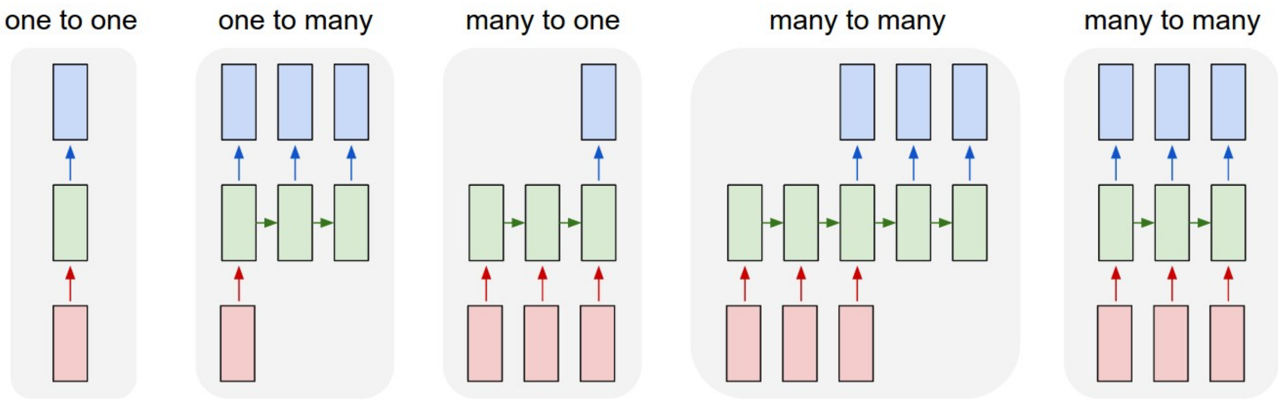
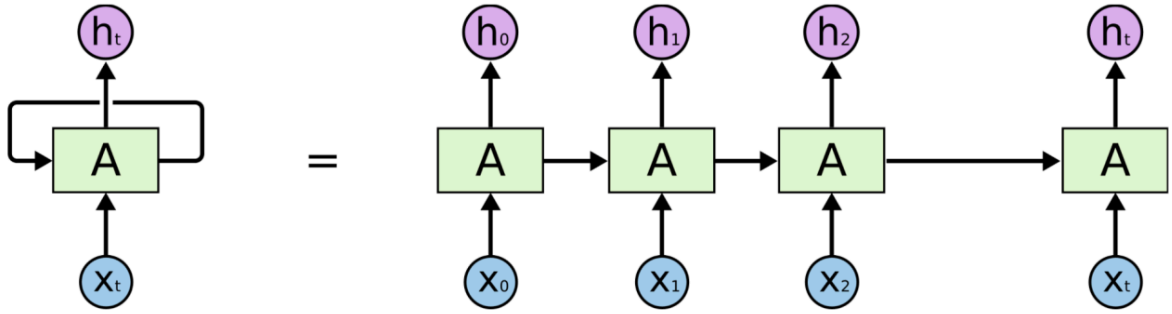


Image from `http://karpathy.github.io/2015/05/21/rnn-effectiveness/`

Why Not a Standard Feed Forward Network?

- For a task such as "Named Entity Recognition" a MLP would have several disadvantages
  - The inputs and outputs may have varying lengths
  - The features wouldn't be shared across different temporal positions in the network
    - Note that 1-D convolutions can be (and are) used to address this, in addition to RNNs - more on this in a later lecture
- To interpret a sentence, or to predict tomorrows weather it is necessary to remember what happened in the past
- To facilitate this we would like to add a feedback loop delayed in time

Recurrent Neural Networks



- RNNs are a family of ANNs for processing sequential data
- RNNs have directed cycles in their computational graphs

Image taken from `https://towardsdatascience.com`

Recurrent Neural Networks

RNNs combine two properties which make them very powerful.

1. Distributed hidden state that allows them to store a lot of information about the past efficiently. This is because several different units can be active at once, allowing them to remember several things at once.
2. Non-linear dynamics that allows them to update their hidden state in complicated ways[1].

[1] Often said to be difficult to train, but this is not necessarily true - dropout can help with overfitting for example

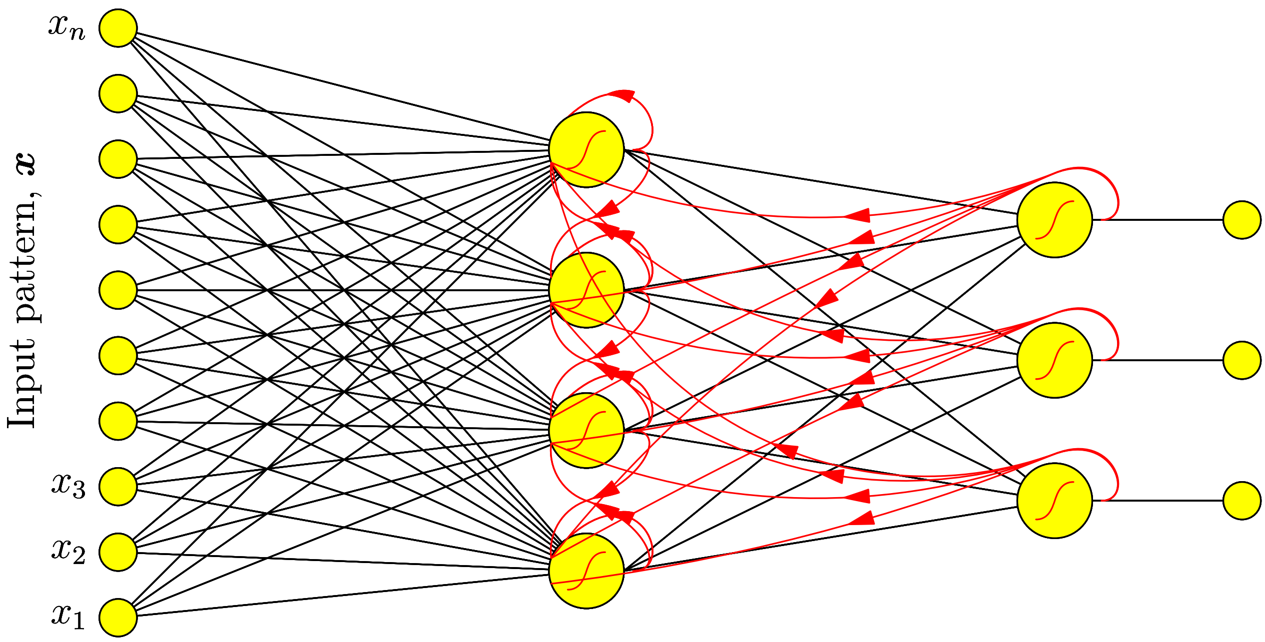
Recurrent Neural Networks

RNNs are Turing complete in the sense they can simulate arbitrary programs[2].

> If training vanilla neural nets is optimisation over functions, training recurrent nets is optimisation over programs.

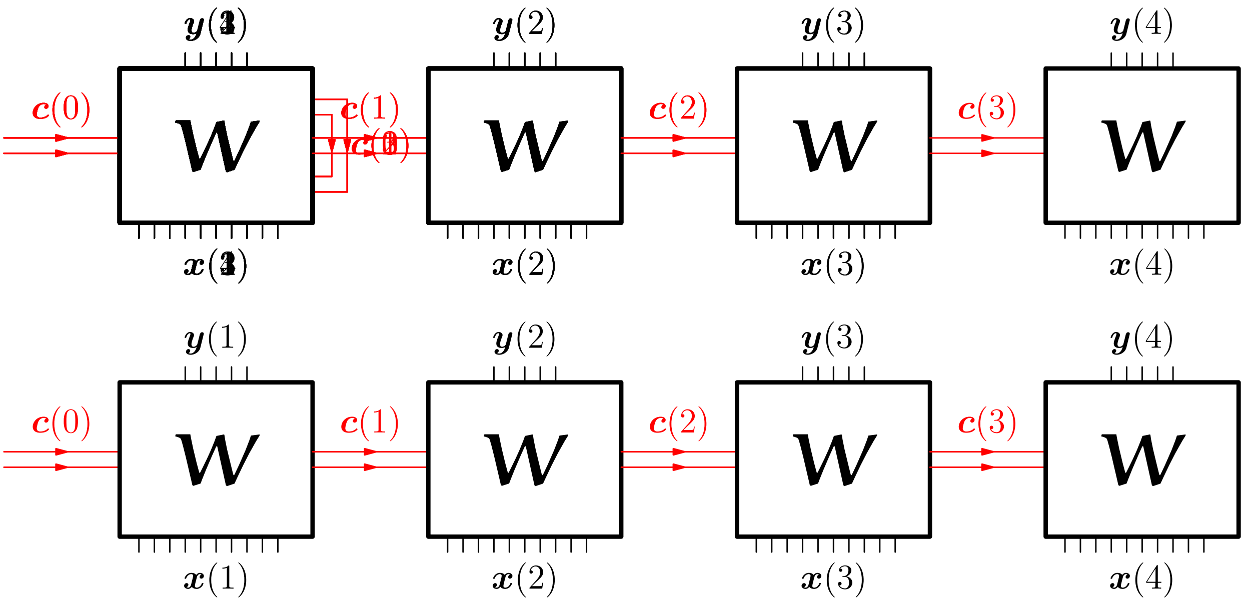
[2] Don't read too much into this - like universal approximation theory, just because they can doesn't mean its necessarily learnable!

Recurrent Network



Training Recurrent Networks

- Given a set of inputs $\mathcal{D} = \big((\boldsymbol{x}(t), \boldsymbol{y}(t)) \big| t = 1, 2, \ldots, T\big)$



- Minimise an error (here MSE, but your choice):

$$E(\boldsymbol{W}) = \sum_{t=1}^{T} \|\boldsymbol{y}(t) - \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W})\|^2$$

- This is known as *back propagation through time*

An RNN is just a recursive function invocation

- $\boldsymbol{y}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W}_f)$
- and the state $\boldsymbol{c}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W}_g)$
- If the output $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-2)$, then prediction will be

$$\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{g}(\boldsymbol{x}(t-1), \boldsymbol{g}(\boldsymbol{x}(t-2), \boldsymbol{g}(\boldsymbol{x}(t-3)|\boldsymbol{W}_g)|\boldsymbol{W}_g)|\boldsymbol{W}_g)|\boldsymbol{W}_f)$$

- it should be clear that the gradients of this with respect to the weights can be found with the chain rule

What is the state update $g()$?

- It depends on the variant of the RNN!
  - Elman
  - Jordan
  - LSTM
  - GRU

Elman Networks ("Vanilla RNNs")

$$\begin{aligned} \boldsymbol{h}_t &= \sigma_h(\boldsymbol{W}_{ih}\boldsymbol{x}_t + \boldsymbol{b}_{ih} + \boldsymbol{W}_{hh}\boldsymbol{h}_{t-1} + \boldsymbol{b}_{hh}) \\ \boldsymbol{y}_t &= \sigma_y(\boldsymbol{W}_y\boldsymbol{h}_t + \boldsymbol{b}_y) \end{aligned}$$

- σ_h is usually tanh
- σ_y is usually identity (linear) – the y's could be regressed values or logits
- the state $\boldsymbol{h}_t$ is referred to as the "hidden state"
- the output at time t is a projection of the hidden state at that time
- the hidden state at time t is a summation of a projection of the input and a projection of the previous hidden state

Going deep: Stacking RNNs

- RNNs can be trivially stacked into deeper networks
- It's just function composition:

$$\boldsymbol{y}(t) = \boldsymbol{f}_2(\boldsymbol{f}_1(\boldsymbol{x}(t), \boldsymbol{c}_2(t-1)|\boldsymbol{W}_1), \boldsymbol{c}_2(t-1)|\boldsymbol{W}_2)$$

- The output of the inner RNN at time t is fed into the input of the outer RNN which produces the prediction y
- Also note: RNNs are most often not used in isolation - it's quite common to process the inputs and outputs with MLPs (or even convolutions)

Example: Character-level language modelling

- We'll end with an example: an RNN that learns to 'generate' English text by learning to predict the next character in a sequence
- This is "Character-level Language Modelling"

Example: Character-level language modelling

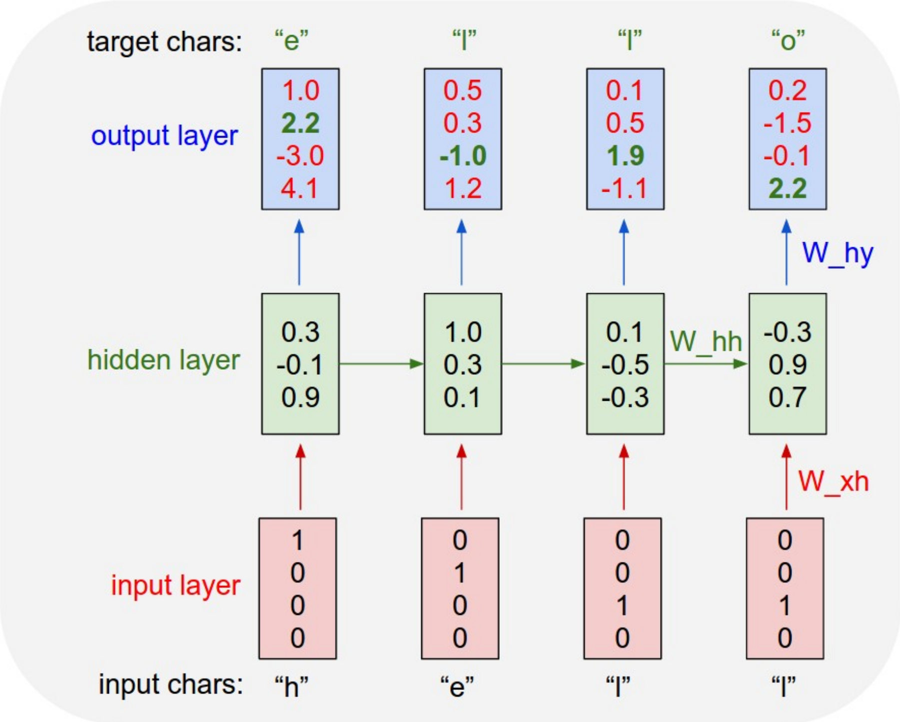


Image from `http://karpathy.github.io/2015/05/21/rnn-effectiveness/`

Training a Char-RNN

- The training data is just text data (e.g. sequences of characters)
- The task is unsupervised (or rather self-supervised): given the previous characters predict the next one
  - All you need to do is train on a reasonable sized corpus of text
  - Overfitting could be a problem: dropout is very useful here

Sampling the Language Model

- Once the model is trained what can you do with it?
- if you feed it an initial character it will output the logits of the next character
- you can use the logits to select the next character and feed that in as the input character for the next timestep
- how do you 'sample' a character from the logits?
  - you could pick the most likely (maximum-likelihood solution), but this might lead to generated text with very low variance (it might be boring and repetitive)
  - you could treat the softmax probabilities defined by the logits as a categorical distribution and sample from them
    - you might increase the 'temperature', T, of the softmax to make the distribution more diverse (less 'peaky'): $q_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$

A lot of the ideas in this lectu on the input c(t - 1), g (x i (x(t - 2), g (t - 1)—W)
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- Sampled from a single layer RNN[3].

[3] LSTM, 128 dim hidden size, with linear input projection to 8-dimensions and output to the number of characters (84). Trained on the text of these slides for 50 epochs.