# Process Sequences

VLC = ∭ Vision Learning Control dVdLdC

# Recurrent Neural Networks

Jonathon Hare

Vision, Learning and Control
University of Southampton

A lot of the ideas in this lecture come from Andrej Karpathy's blog post on the Unreasonable Effectiveness of RNNs (http://karpathy.github.io/2015/05/21/rnn-effectiveness/). Many of the images and animations were made by Adam Prügel-Bennett.

# Recurrent Neural Networks - Motivation

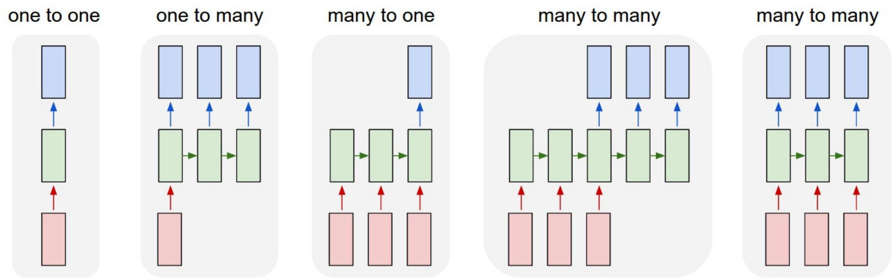
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$: | Jon | and | Antonia | gave | deep | learning | lectures |
| $y$: | 1 | 0 | 1 | 0 | 0 | 0 | 0 |

# Recurrent Neural Networks - Motivation

| | | | | | |
|---|---|---|---|---|---|
| $x$: | $x^{(1)}$ | ... | $x^{(t)}$ | ... | $x^{(T_x)}$ |
| $x$: | Jon | ... | Antonia | ... | lectures |
| | | | | | |
| $y$: | $y^{(1)}$ | ... | $y^{(t)}$ | ... | $y^{(T_y)}$ |
| $y$: | 1 | ... | 1 | ... | 0 |

In this example, $T_x = T_y = 7$ but $T_x$ and $T_y$ can be different.

# Recurrent Neural Networks

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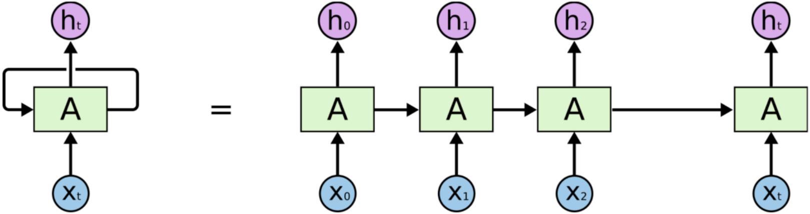
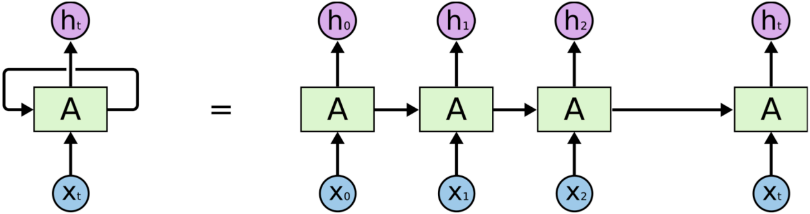
- To facilitate this we would like to add a feedback loop delayed in time

# Recurrent Neural Networks




- RNNs are a family of ANNs for processing sequential data

Image taken from `https://towardsdatascience.com`

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- RNNs have directed cycles in their computational graphs

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# Recurrent Neural Networks

RNNs combine two properties which make them very powerful.

1. Distributed hidden state that allows them to store a lot of information about the past efficiently. This is because several different units can be active at once, allowing them to remember several things at once.
2. Non-linear dynamics that allows them to update their hidden state in complicated ways[1].

[1] Often said to be difficult to train, but this is not necessarily true - dropout can help with overfitting for example

# Recurrent Neural Networks

RNNs are Turing complete in the sense they can simulate arbitrary programs[2].

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If training vanilla neural nets is optimisation over functions, training recurrent nets is optimisation over programs.

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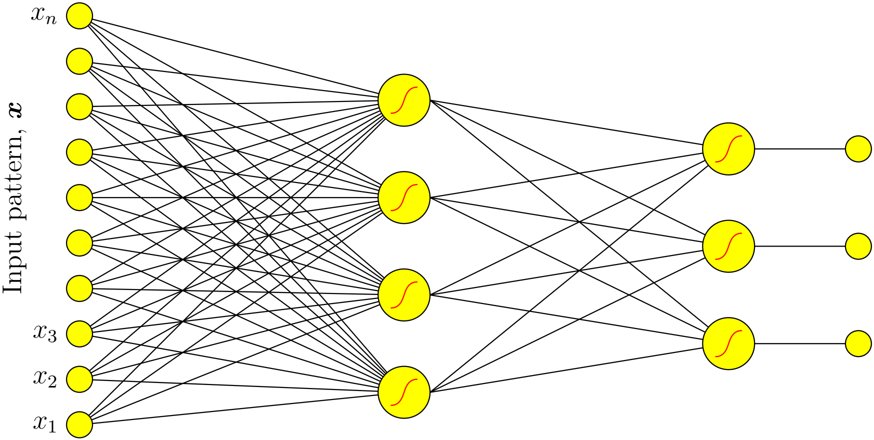
# Recurrent Network



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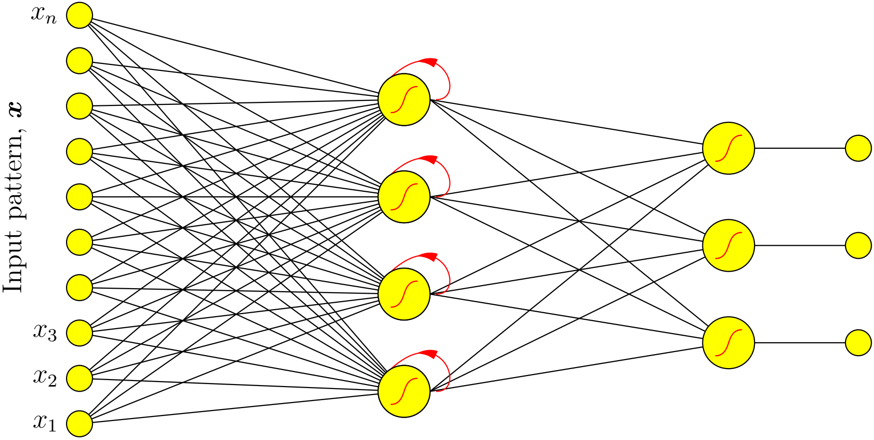

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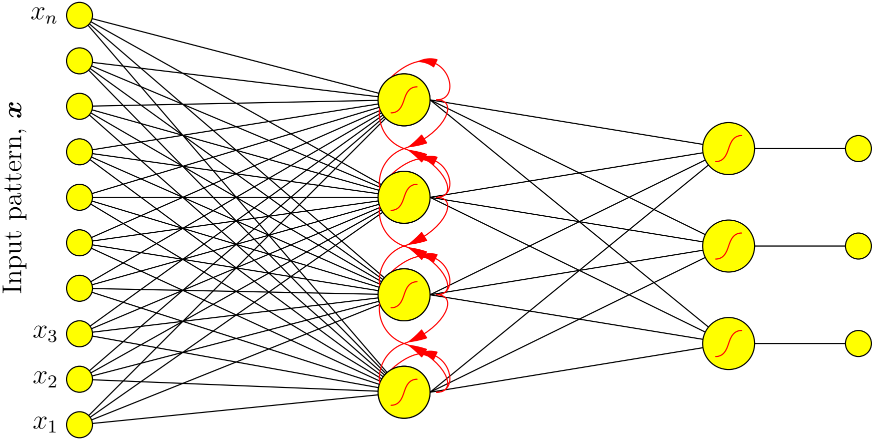

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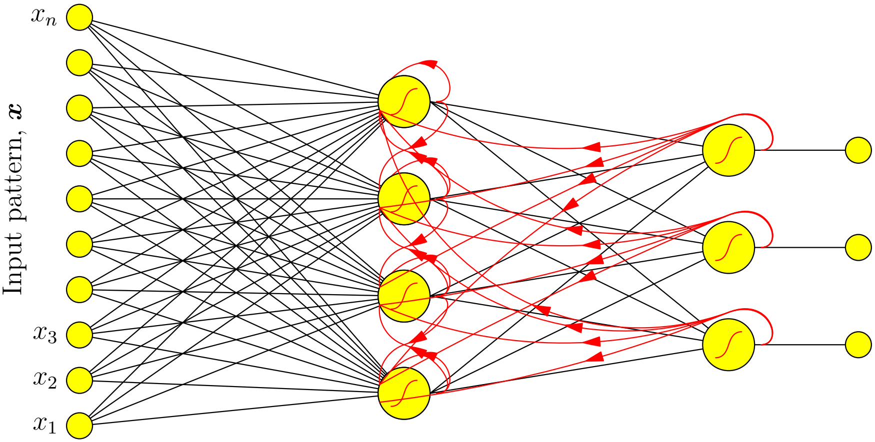

# Training Recurrent Networks

- Given a set of inputs $\mathcal{D} = \big((\boldsymbol{x}(t), \boldsymbol{y}(t))\big| t = 1, 2, \ldots, T\big)$

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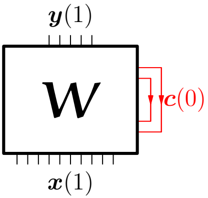


- Minimise an error (here MSE, but your choice):

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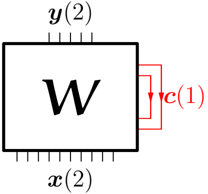


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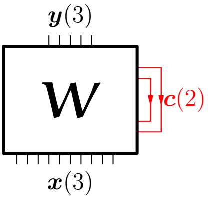


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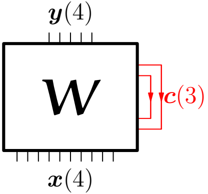


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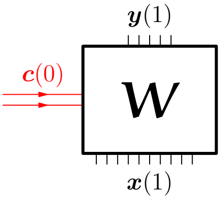


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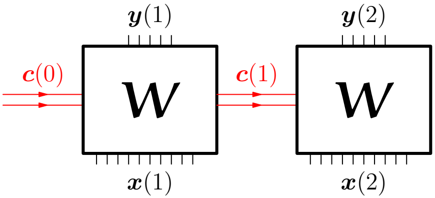


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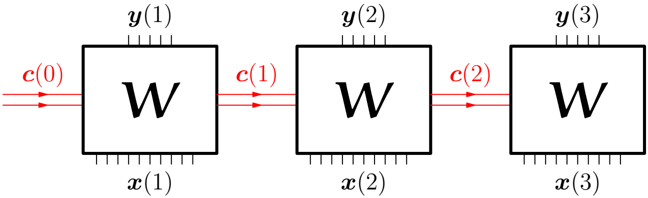


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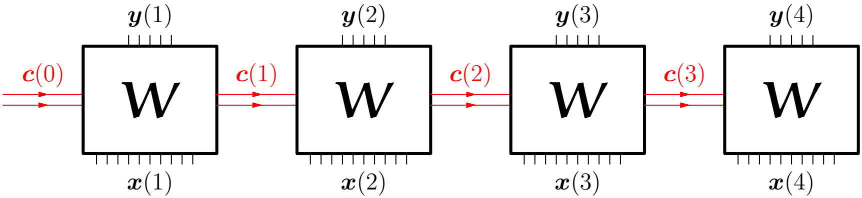


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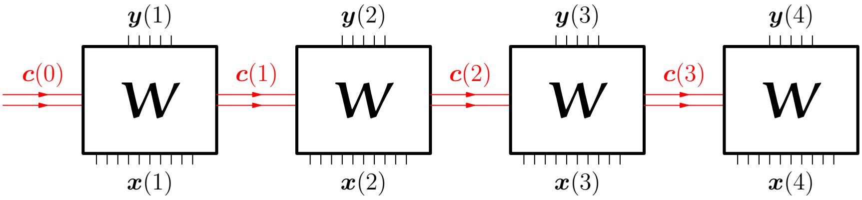


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- This is known as *back-propagation through time*

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- it should be clear that the gradients of this with respect to the weights can be found with the chain rule

# What is the state update $g()$?

- It depends on the variant of the RNN!
  - Elman
  - Jordan
  - LSTM
  - GRU

# Elman Networks ("Vanilla RNNs")

$$\boldsymbol{h}_t = \sigma_h(\boldsymbol{W}_{ih}\boldsymbol{x}_t + \boldsymbol{b}_{ih} + \boldsymbol{W}_{hh}\boldsymbol{h}_{t-1} + \boldsymbol{b}_{hh})$$
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- the hidden state at time $t$ is a summation of a projection of the input and a projection of the previous hidden state

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- The output of the inner RNN at time $t$ is fed into the input of the outer RNN which produces the prediction $y$
- Also note: RNNs are most often not used in isolation - it's quite common to process the inputs and outputs with MLPs (or even convolutions)

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- We'll end with an example: an RNN that learns to 'generate' English text by learning to predict the next character in a sequence

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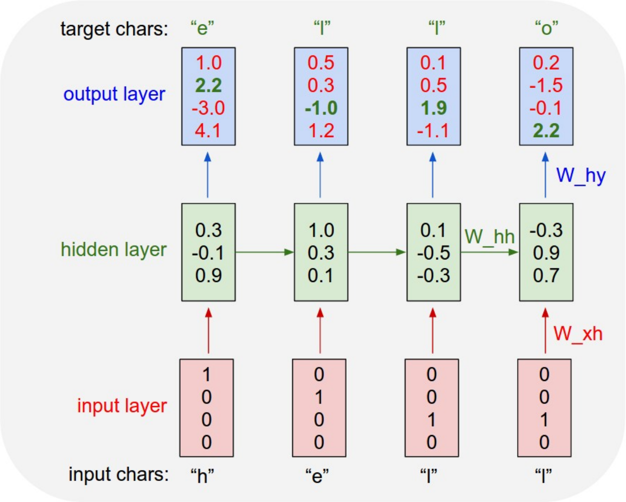


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# Training a Char-RNN

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  - Overfitting could be a problem: dropout is very useful here

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    - you could pick the most likely (maximum-likelihood solution), but this might lead to generated text with very low variance (it might be boring and repetitive)
    - you could treat the softmax probabilities defined by the logits as a categorical distribution and sample from them
        - you might increase the 'temperature', $T$, of the softmax to make the distribution more diverse (less 'peaky'): $q_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$

*A lot of the ideas in this lectu on the input c(t - 1), g (x i (x(t - 2), g (t - 1)—W )*
*Iged snllhomitpon" ares Mnt Net) th*
*pl*
*Onaafed a tre the sidisicters of to prediction couponet on the logits its venvows usts sevouvd be this in as useuled at on in the pan Lerate'atectsrray to paet inputs D = Pxxpraition the rople, the next*
*vog the state atite*

- Sampled from a single layer RNN[3].

[3]LSTM, 128 dim hidden size, with linear input projection to 8-dimensions and output to the number of characters (84). Trained on the text of these slides for 50 epochs.