

**Forget to remember**  
**Remember to forget**

# Long Short Term Memories and Gated Recurrent Units

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Vision, Learning and Control  
University of Southampton

Some of the images and animations used here were originally designed by Adam Prügel-Bennett.

## Recap: An RNN is just a recursive function invocation

- $\mathbf{y}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{c}(t) | \mathbf{W}_f)$
- and the state  $\mathbf{c}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{c}(t-1) | \mathbf{W}_g)$
- If the output  $\mathbf{y}(t)$  depends on the input  $\mathbf{x}(t-2)$ , then prediction will be

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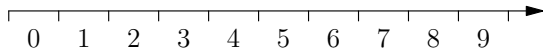
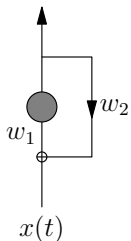
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- If  $\mathbf{y}(t)$  depends on the input  $\mathbf{x}(t-\tau)$  then the back-propagated signal will be proportional to  $a^{\tau-1}$
- This either vanishes or explodes when  $\tau$  becomes large

# Vanishing and Exploding Gradients

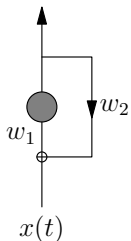
$$y(t) = w_1 (x(t) + w_2 y(t - 1))$$



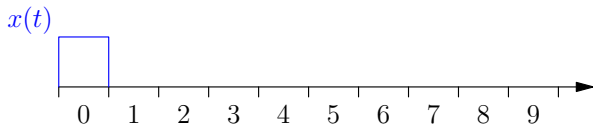


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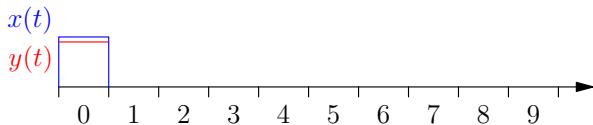
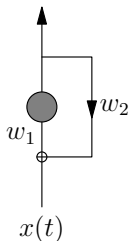
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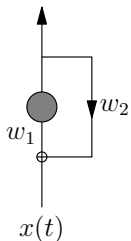
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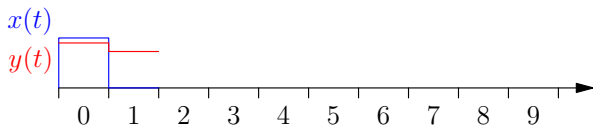


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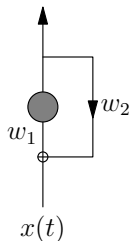


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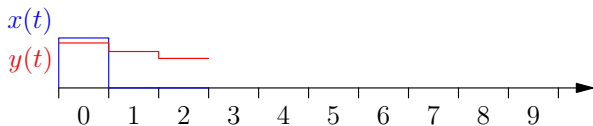


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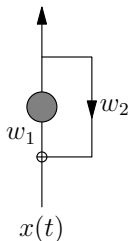


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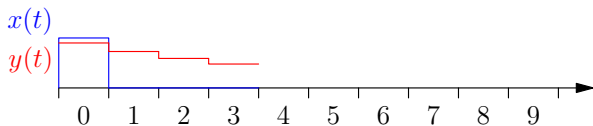


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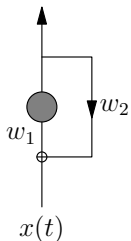


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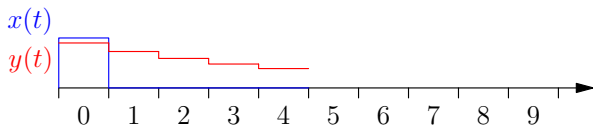


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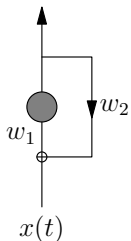


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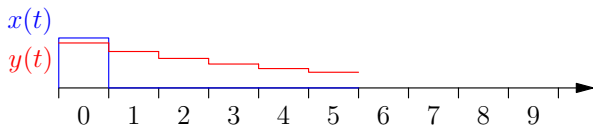


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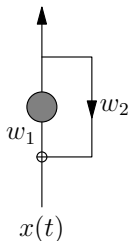


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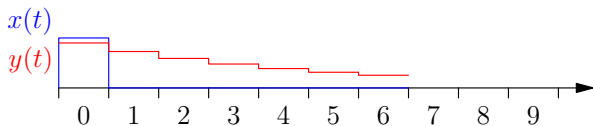


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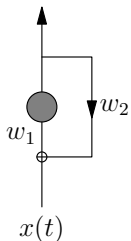
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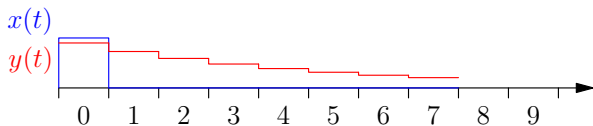


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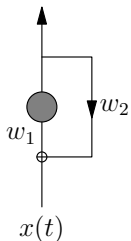


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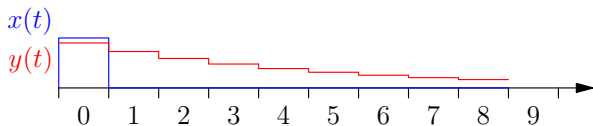


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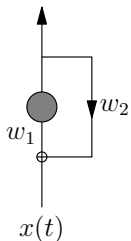


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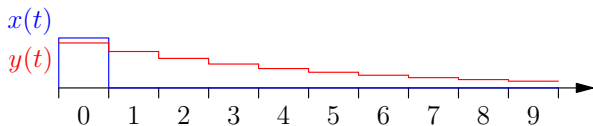


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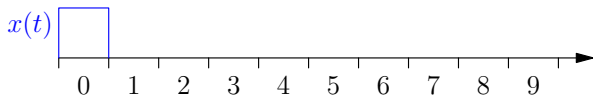
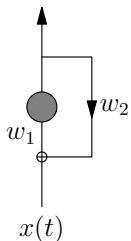
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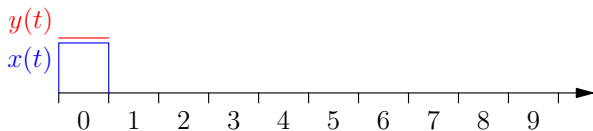
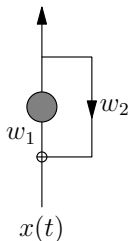
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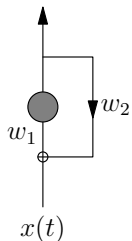
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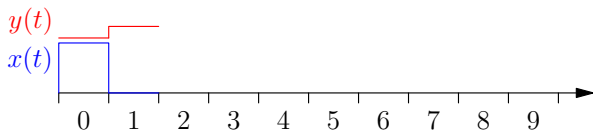


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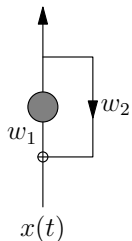


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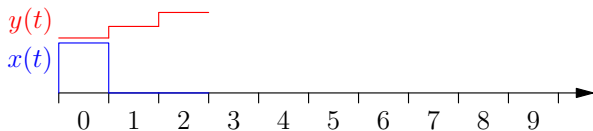


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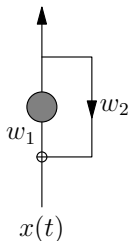


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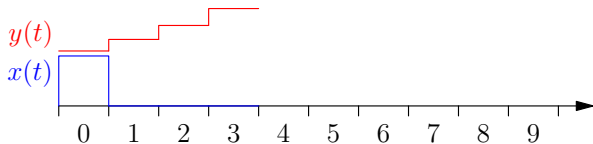


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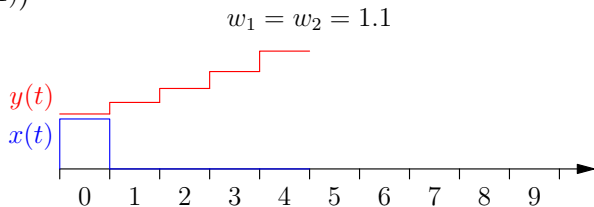
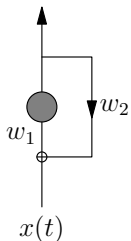
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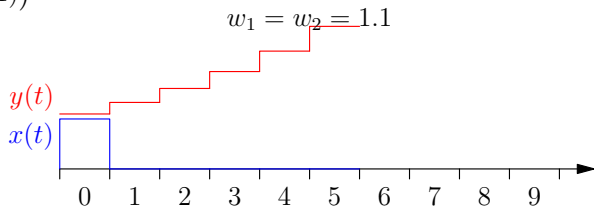
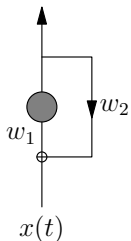
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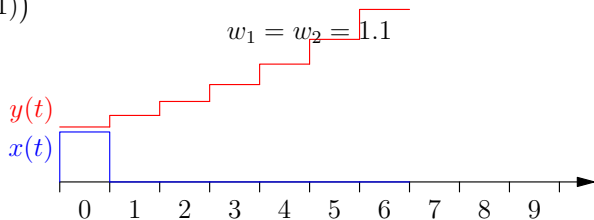
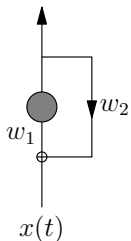
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- To do this we should use 'gates' that saturate at 0 and 1
- Sigmoid functions naturally saturate at 0 and 1

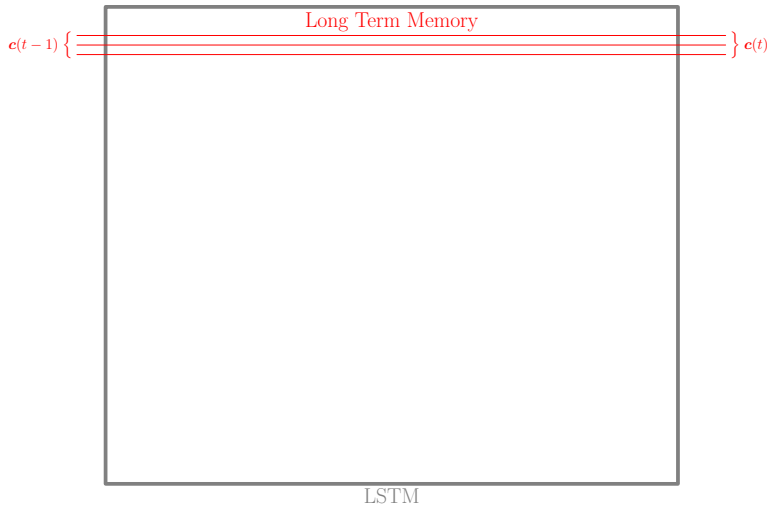


# LSTM Architecture

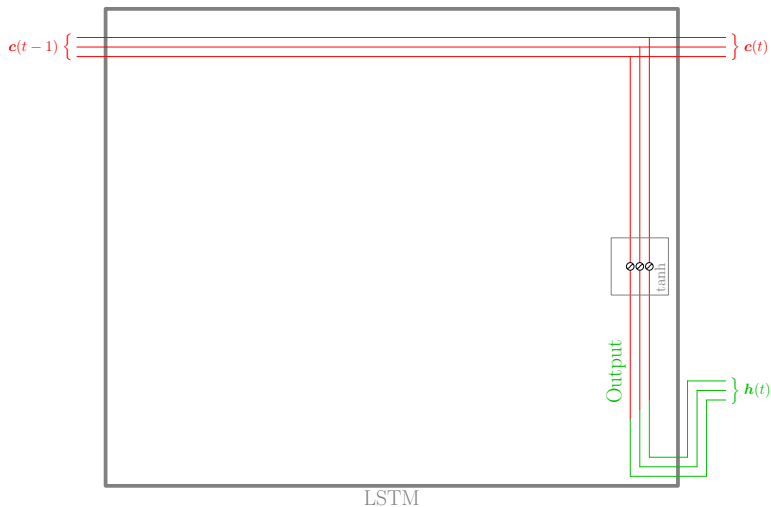


LSTM

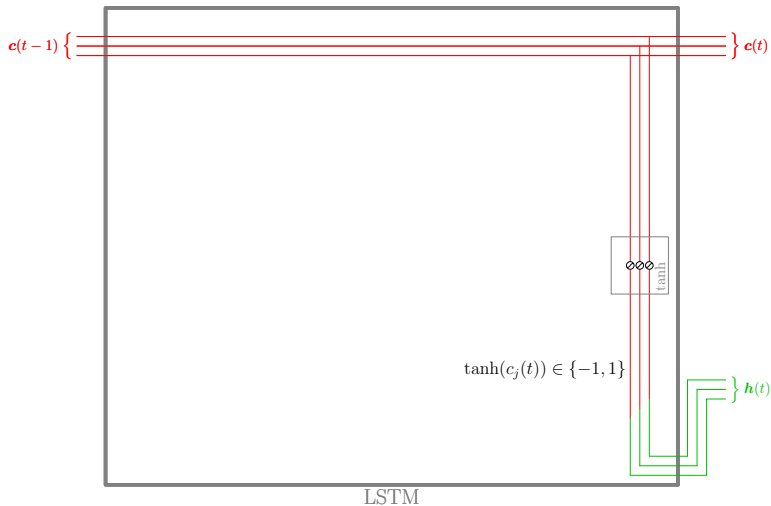
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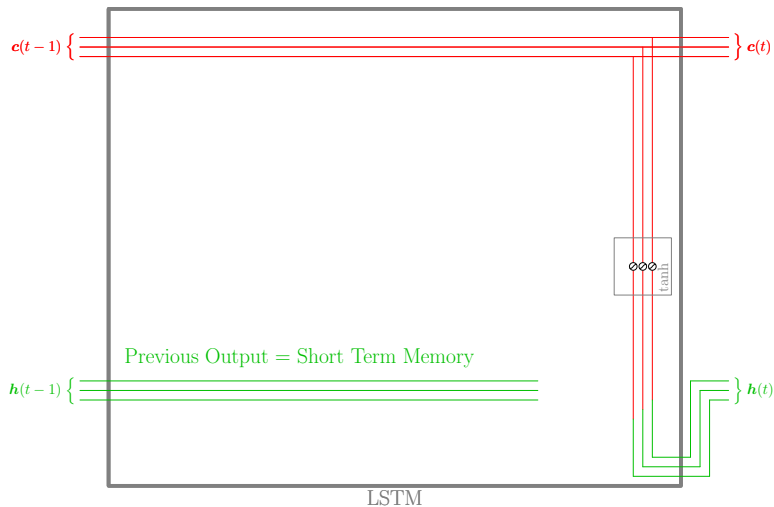
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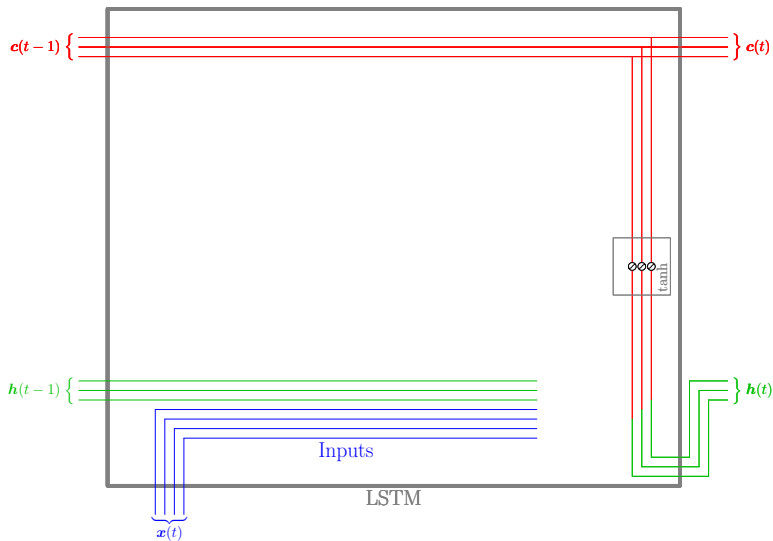
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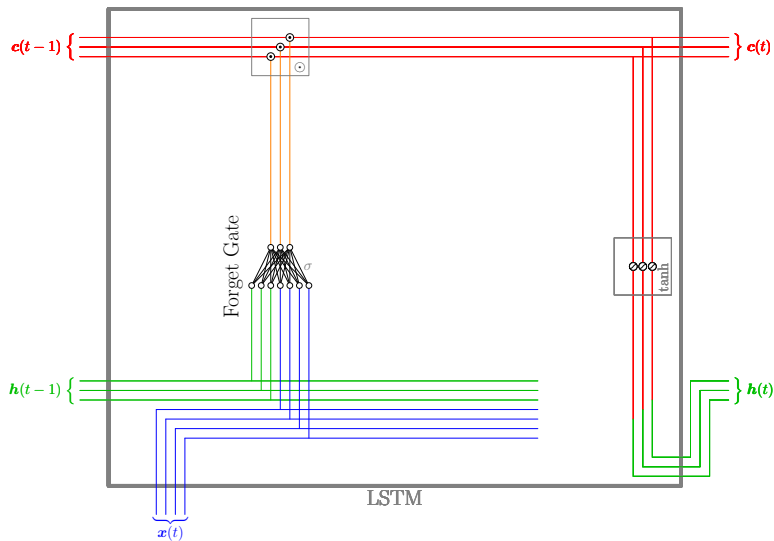
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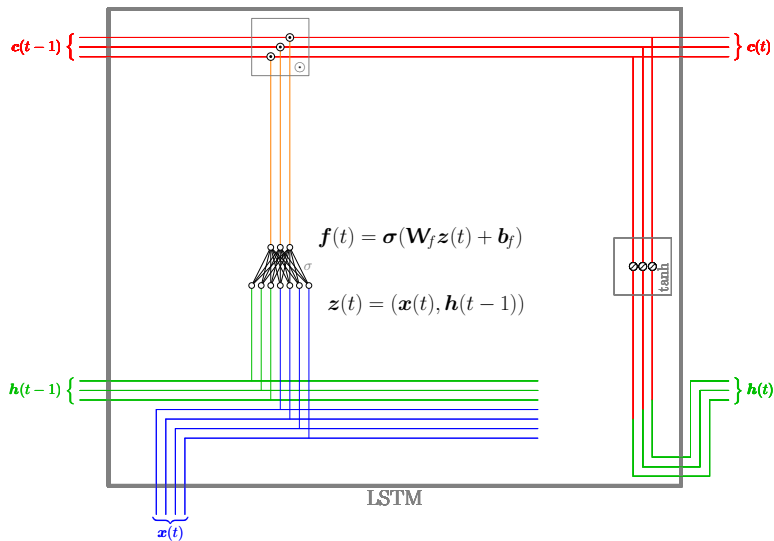
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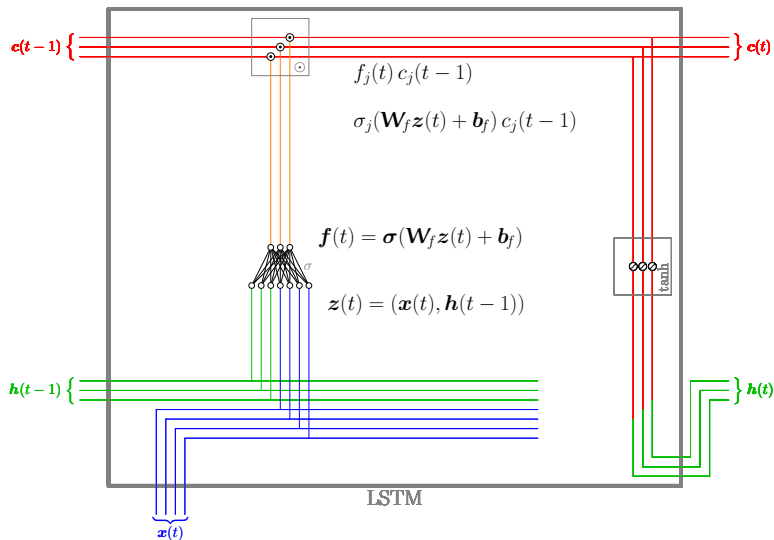


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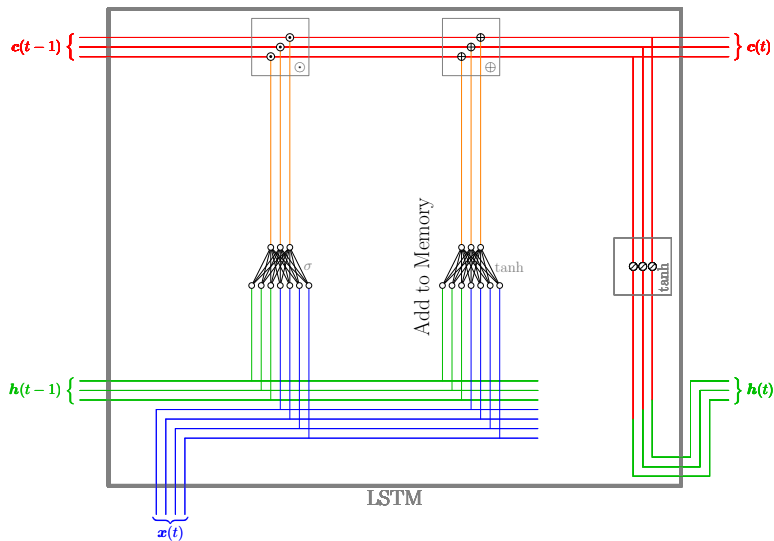




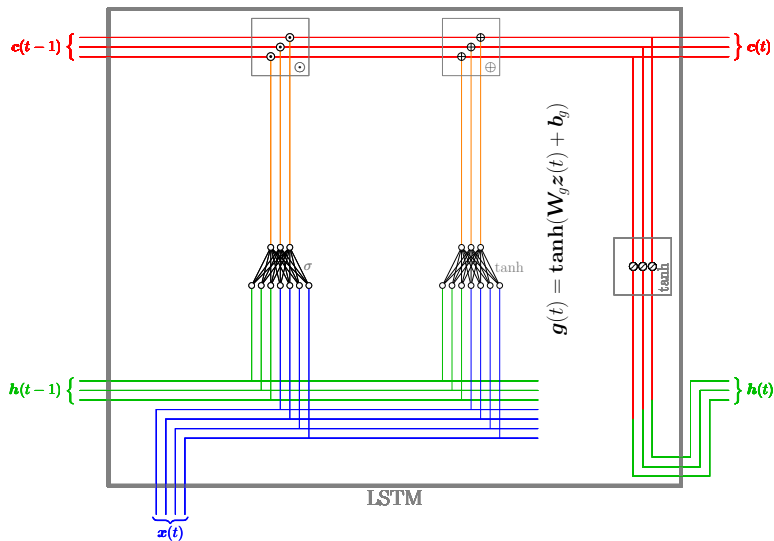
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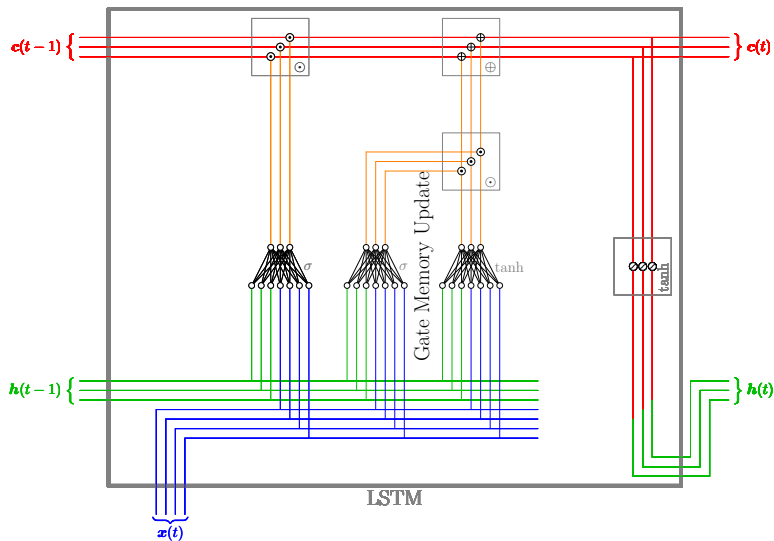
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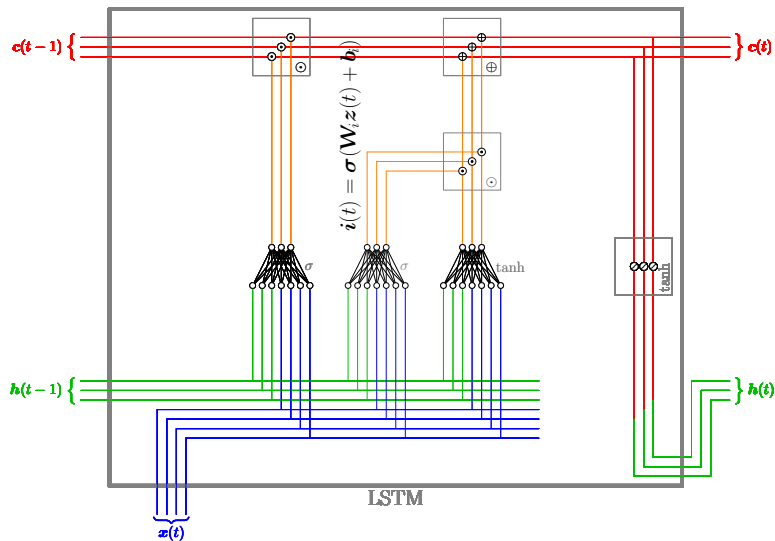
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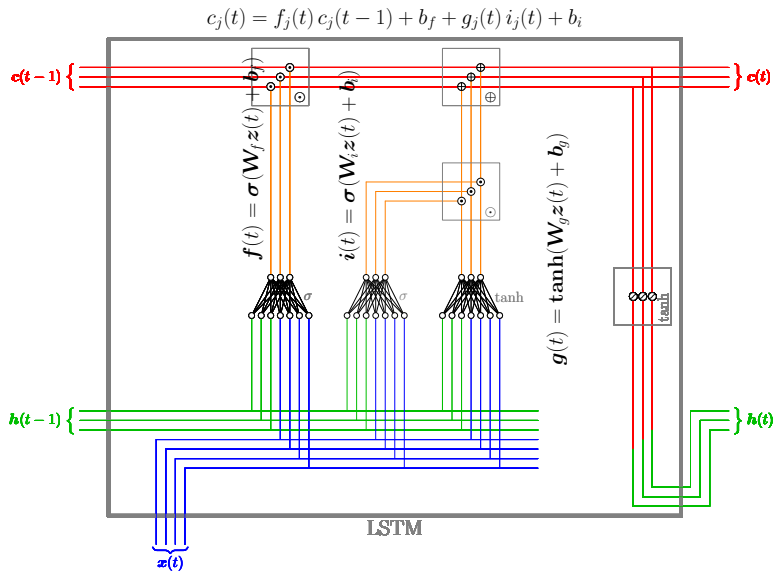
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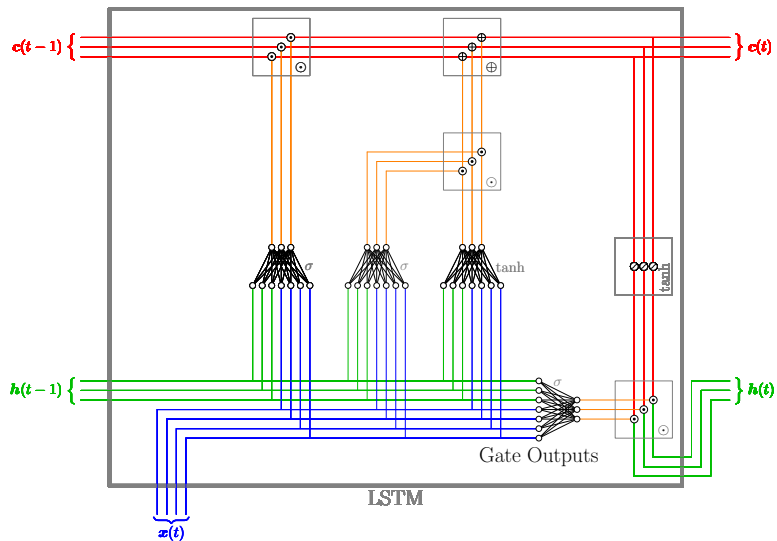
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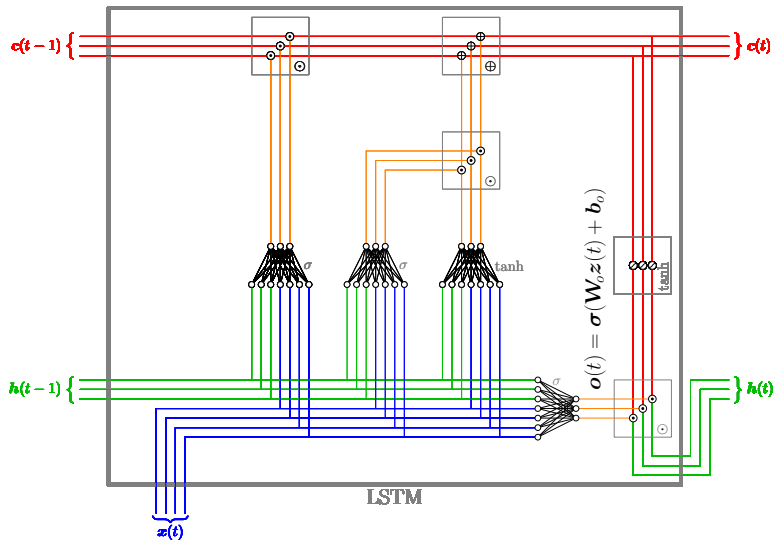
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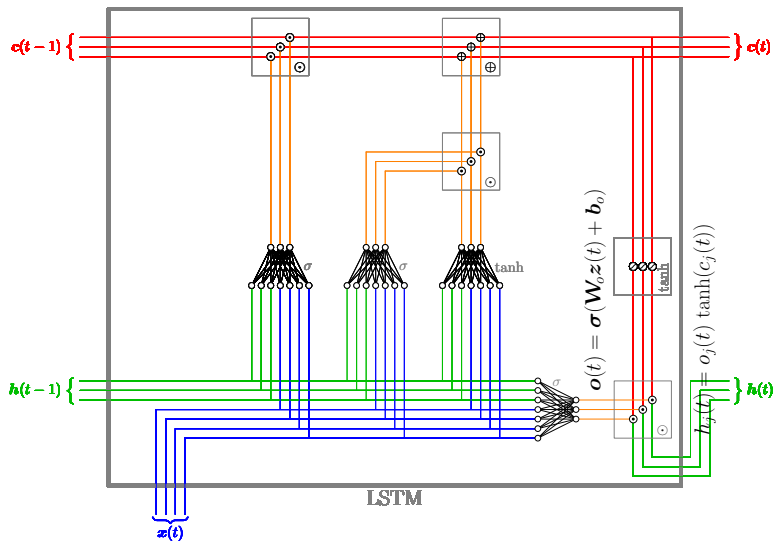


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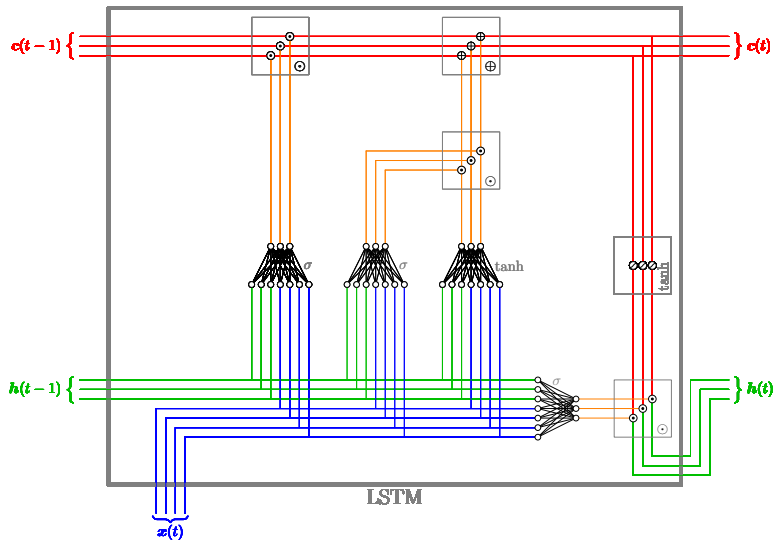




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- Inputs  $\mathbf{z}(t) = (\mathbf{x}(t), \mathbf{h}(t - 1))$
- Network updates ( $\mathbf{W}_*$  and  $\mathbf{b}_*$  are the learnable parameters)

$$\begin{aligned} \mathbf{f}(t) &= \sigma(\mathbf{W}_f \mathbf{z}(t) + \mathbf{b}_f) & \mathbf{i}(t) &= \sigma(\mathbf{W}_i \mathbf{z}(t) + \mathbf{b}_i) \\ \mathbf{g}(t) &= \tanh(\mathbf{W}_g \mathbf{z}(t) + \mathbf{b}_g) & \mathbf{o}(t) &= \sigma(\mathbf{W}_o \mathbf{z}(t) + \mathbf{b}_o) \end{aligned}$$

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Initially, for  $t = 1$ ,  $\mathbf{h}(0) = \mathbf{0}$

- Inputs  $\mathbf{z}(t) = (\mathbf{x}(t), \mathbf{h}(t-1))$
- Network updates ( $\mathbf{W}_*$  and  $\mathbf{b}_*$  are the learnable parameters)

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$$\mathbf{c}(t) = \mathbf{f}(t) \odot \mathbf{c}(t-1) + \mathbf{g}(t) \odot \mathbf{i}(t)$$

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- Output  $\mathbf{h}(t) = \mathbf{o}(t) \odot \tanh(\mathbf{c}(t))$

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# Training LSTMs

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- Note that it involves four dense layers with sigmoidal (or tanh) outputs.
- This means that typically it is very slow to train.
- There are a few variants of LSTMs, but all are very similar. The most popular is probably the Gated Recurrent Unit (GRU).

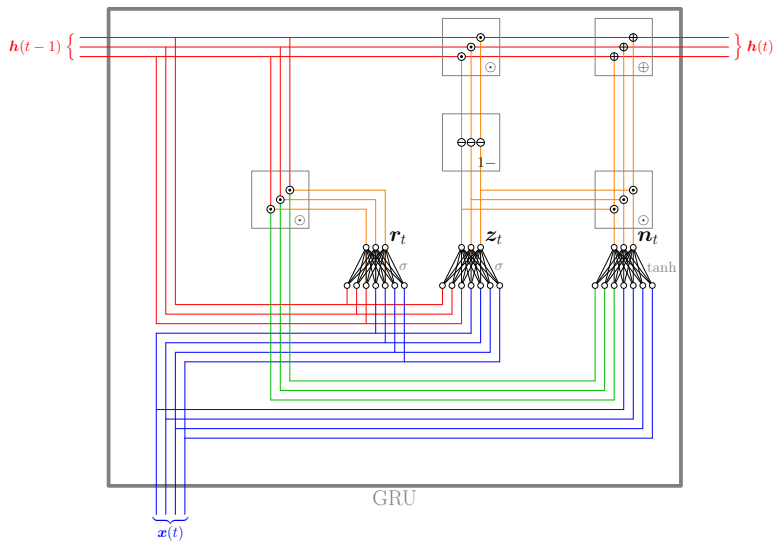
# LSTM Success Stories

- LSTMs have been used to win many competitions in speech and handwriting recognition.
- Major technology companies including Google, Apple, and Microsoft are using LSTMs as fundamental components in products.
- Google used LSTM for speech recognition on the smartphone, for Google Translate.
- Apple uses LSTM for the "Quicktype" function on the iPhone and for Siri.
- Amazon uses LSTM for Amazon Alexa.
- In 2017, Facebook performed some 4.5 billion automatic translations every day using long short-term memory networks<sup>1</sup>.

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<sup>1</sup>[https://en.wikipedia.org/wiki/Long\\_short-term\\_memory](https://en.wikipedia.org/wiki/Long_short-term_memory)

# Gated Recurrent Unit (GRU)



# Gated Recurrent Unit (GRU)

- $\mathbf{x}(t)$ : input vector
- $\mathbf{h}(t)$ : output vector (and 'hidden state')
- $\mathbf{r}(t)$ : reset gate vector
- $\mathbf{z}(t)$ : update gate vector
- $\mathbf{n}(t)$ : new state vector (before update is applied)
- $\mathbf{W}$  and  $\mathbf{b}$ : parameter matrices and biases

# Gated Recurrent Unit (GRU)

Initially, for  $t = 1$ ,  $\mathbf{h}(0) = \mathbf{0}$

$$\mathbf{z}(t) = \sigma(\mathbf{W}_z(\mathbf{x}(t), \mathbf{h}(t-1)) + \mathbf{b}_z)$$

$$\mathbf{r}(t) = \sigma(\mathbf{W}_r(\mathbf{x}(t), \mathbf{h}(t-1)) + \mathbf{b}_r)$$

$$\mathbf{n}(t) = \tanh(\mathbf{W}_n(\mathbf{x}(t), \mathbf{r}(t) \odot \mathbf{h}(t-1)) + \mathbf{b}_n)$$

$$\mathbf{h}(t) = (1 - \mathbf{z}(t)) \odot \mathbf{h}(t-1) + \mathbf{z}(t) \odot \mathbf{n}(t)$$

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Most implementations follow the original paper and swap  $(1 - \mathbf{z}(t))$  and  $(\mathbf{z}(t))$  in the  $\mathbf{h}(t)$  update; this doesn't change the operation of the network, but does change the interpretation of the update gate, as the gate would have to produce a 0 when an update was to occur, and a 1 when no update is to happen (which is somewhat counter-intuitive)!

# GRU or LSTM?

- GRUs have two gates (reset and update) whereas LSTM has three gates (input/output/forget)
- GRU performance on par with LSTM but computationally more efficient (less operations & weights).
- In general, if you have a very large dataset then LSTMs will likely perform slightly better.
- GRUs are a good choice for smaller datasets.